Drift-Wave Turbulence

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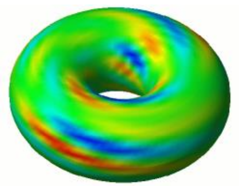

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Outline

I. Drift Wave Mechanism

- Drift-Wave mechanism from density gradients
- Breaking the $E_{||}$=0 constraint and resistive drift-waves
- Drift waves in the laboratory

II. Gradient Energy Released to Drift Waves and Critical Gradients

- Density gradients and temperature gradients as turbulence sources
- Conditions for transport and propagation of disturbances
- Drift-wave as coherent structures including vortices and streamer

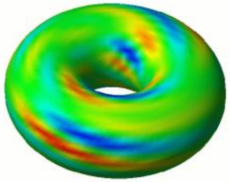
III. Drift-Wave Eigenmodes in Toroidal Geometry

- Trapped Particle Instability and the TEM modes
- Ballooning modes and turbulence simulations
- Reversed shear and Er *shear* Ballooning modes and turbulence simulations

IV. Turbulence Simulations

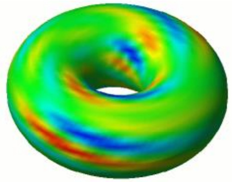
- Inverse cascade and generation of sheared flows
- Ion acoustic wave coupling and dispersion from the polarization current
- Ballooning modes and turbulence simulations
- Role of Magnetic Islands

V. Transport Simulations for large Tokamaks with Turbulence



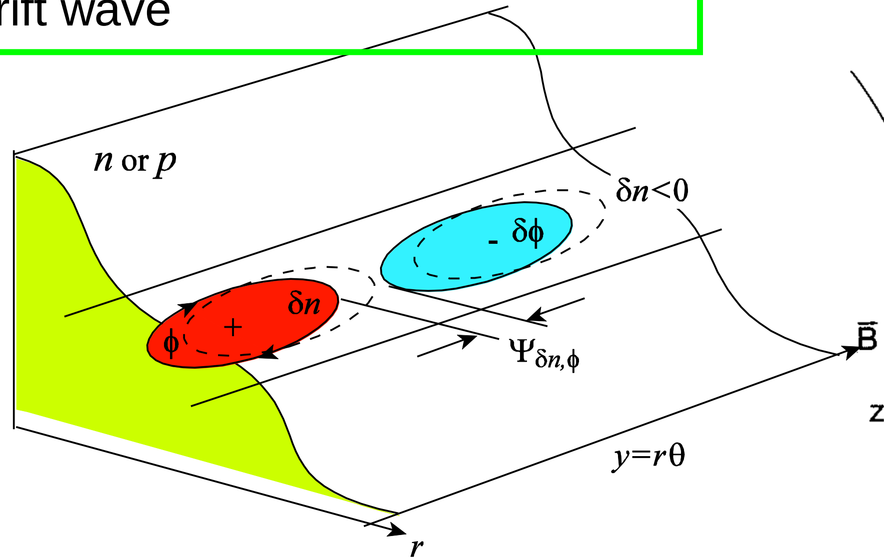
I. Drift Wave Mechanism

- Drift-Wave mechanism from density gradients
- Drift waves in the laboratory
- Breaking the $E_{||}$=0 constraint with resistive drift-waves

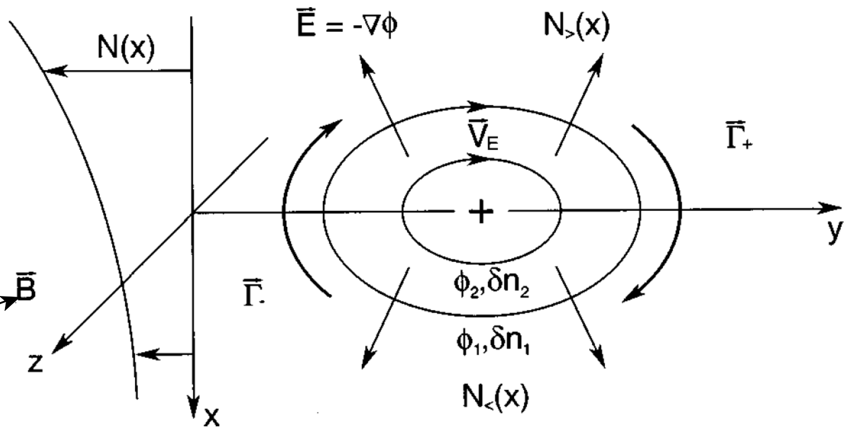


Drift Wave Mechanism

Density or pressure profile with drift wave



Top view with B_z out of page



The potential and density fluctuations are:

- Quasi-neutral
- Small amplitude waves
- Coherent vortex structures form for amplitudes that exceed the EXB trapping threshold

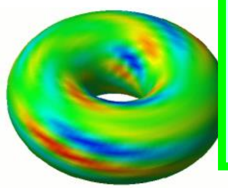
$v_E > v_{de} = (T_e/\text{-}eBn_e)\, dn_e/dx \equiv T_e/eBL_n = (\rho_s/L_n)\, c_s$

$C_s \sim 10^7$ cm/s

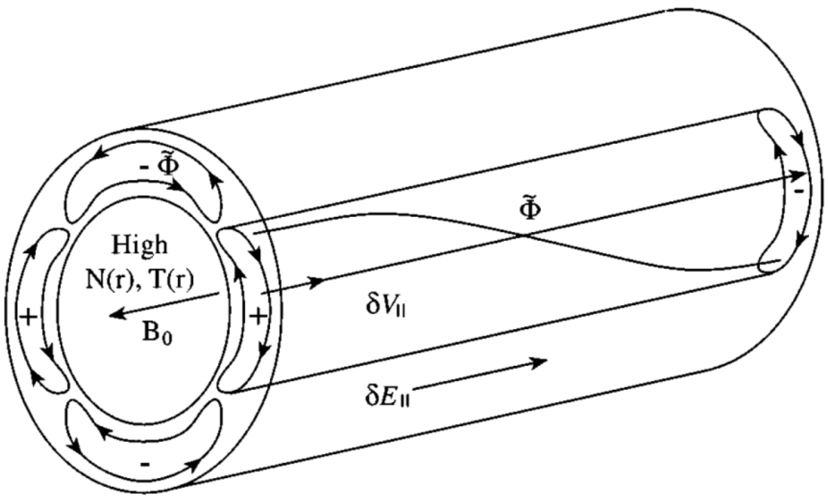
$\rho_s/L_n \sim 1\text{cm}/100\text{cm}$

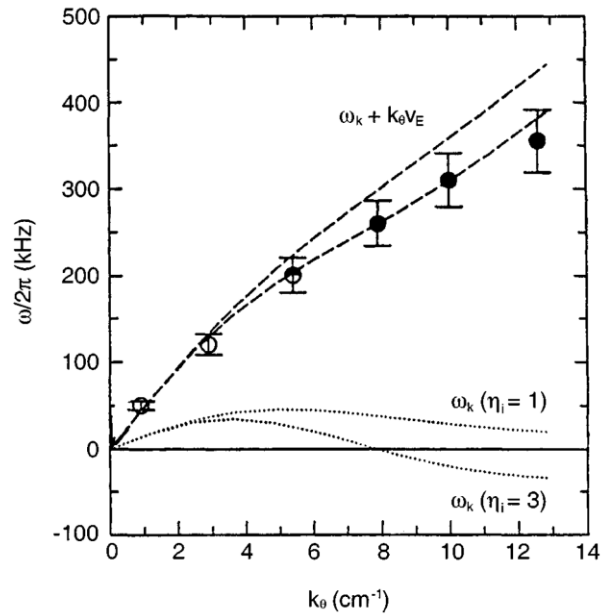
$\omega_{*e} = k_y v_{de} \sim 10^5$ rad/s

$D_{gB} = \rho_s v_{de} \sim 10^4\ \text{cm}^2/\text{s}$



Experiments in Q-device, CLM, LAPD,TEXT





- Measured parallel wavelength $\omega(k) < k_{\|} v_e$
- Frequency and growth rates agree with theory

eg. Hendel et al, Phys. Fluids 11, 2426, 1968 [Q machine]

Sen et al, PRL 66, 429,1991 [CLM]

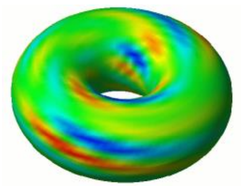
Horton et al, PoP 12, 2005 [LAPD]

Perez et al. PoP 13, 03210, 2006 [LAPD] with velocity shear

Brower et al. PRL 1987 in TEXT with HIBP for E_r

In tokamak the high azimuthal mode numbers are unstable and have phase shifts

Wave-particle interactions or resistivity make the waves grow exponentially $\exp(\gamma_k t)$



Ion Scale Anomalous Transport

- $\mathbf{E}\times\mathbf{B}$ flows from plasma potential $\mathbf{E} = -\nabla\varphi$

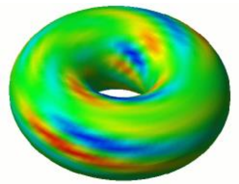
$$\mathbf{v}_E = -\frac{\nabla\varphi \times \mathbf{B}}{B^2} = \frac{\mathbf{e}_z \times \nabla\varphi}{B}$$

- Stream function $\psi(\mathbf{x},t) = \dfrac{\varphi(\mathbf{x},t)}{B}$
- Rotational motion quantified by vorticity

$$\omega = \hat{\mathbf{b}}\cdot\nabla\times\mathbf{v} = \frac{1}{B}\nabla_\perp^2\varphi = \nabla_\perp^2\psi.$$

- $\dfrac{d\omega}{dt}$ is present in nonlinear dynamics of
  - Kelvin-Helmholtz instability (KHI).
  - Interchange modes.
  - Drift waves: Hasegawa-Mima, Hasegawa-Wakatani, Ion Temperature Gradient (ITG), Electron Temperature Gradient (ETG).

Vorticity Probe direct, high-resolution measurement of plasma vorticity [Perez, PoP (2006)]



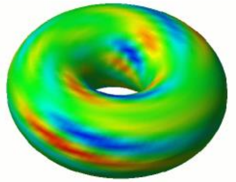
Resistive Drift Waves with $E_\parallel \neq 0$ Driven Unstable by Density Gradients

Close current loops $\nabla \cdot \boldsymbol{j} = 0$ gives for $\boldsymbol{E} = -\nabla\phi$ modes

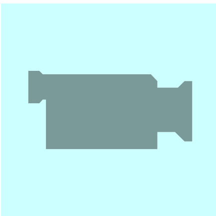
$$\left\{ \frac{m_i n_i}{B^2} \left[(\omega - k_y v_y)(k_y^2 - \partial_x^2) + k_y \frac{d^2 v_y}{dx^2} \right] + \frac{i n_e e^2 k_\parallel^2(x)\,(1 - \omega_{*e}/\omega)}{m_e \left[\nu_e + i k_\parallel^2 v_e^2/\omega \right]} \right\} \phi = 0$$

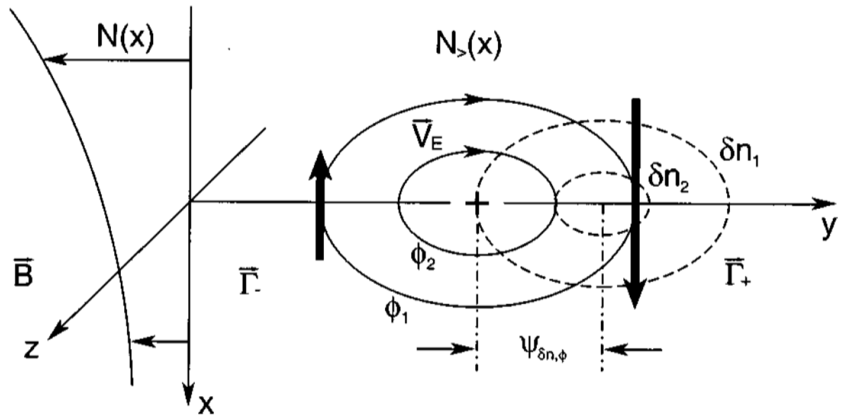
Two Regimes

- ☐ Collisionless drift waves $\frac{k_\parallel^2 v_e^2}{\nu_e} > |\omega|$: Hasegawa-Mima Model
- ☐ Collsiional drift wave $\frac{k_\parallel^2 v_e^2}{\nu_e} < |\omega|$: Hasegawa-Wakatani Model



Breaking the Frozen-in Magnetic Flux





- Ohm's Law

$$E_{\|} + \frac{1}{en_e}\nabla_{\|}p_e = \eta j_{\|} \quad \rightarrow \quad -ik_{\|}\left(\phi - \frac{T_e}{e}\frac{\delta n_e}{n_e}\right) = \eta \delta j_{\|}$$

- Mass Conservation

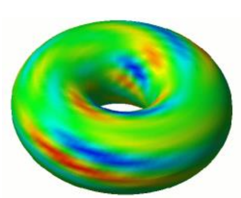
$$\frac{\partial n_i}{\partial t} + \boldsymbol{v}_E \cdot \boldsymbol{\nabla} n_i + n_i \boldsymbol{\nabla}\cdot\boldsymbol{v}_E = 0 \quad \rightarrow \quad -i\omega\delta n_i + \frac{ik_y\phi}{B}\frac{\partial n}{\partial r} + \frac{2nv_{E,x}}{R} = 0$$

- Quasi-Neutrality $\delta n_e = Z_i \delta n_i$ implies $\boldsymbol{\nabla}\cdot\delta\boldsymbol{j} = 0$.

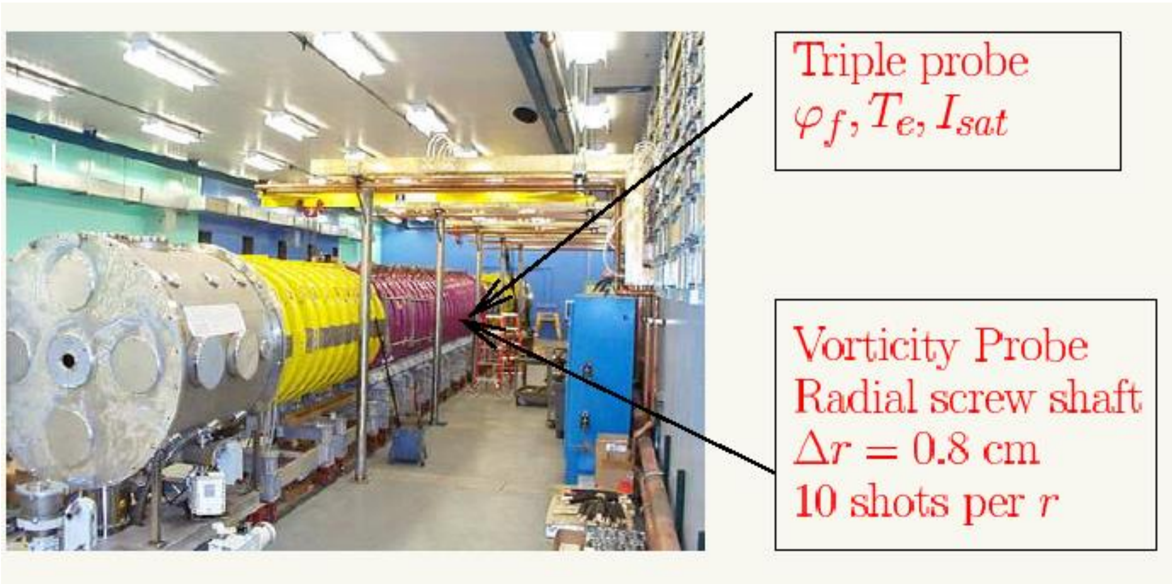
Alfven Wave

$$\left[\frac{m_i n k_\perp^2(\omega - \omega_{*i})}{B^2} - \frac{2k_y^2 p'}{\omega B^2 R}\right]\phi + \frac{B}{\mu_0}\frac{d}{dz}\left[\frac{k_\perp^2}{B}\frac{(1+k_\perp^2\rho_s^2)}{(1+k_\perp^2\delta^2)}\frac{d\phi}{dz}\right] = 0$$

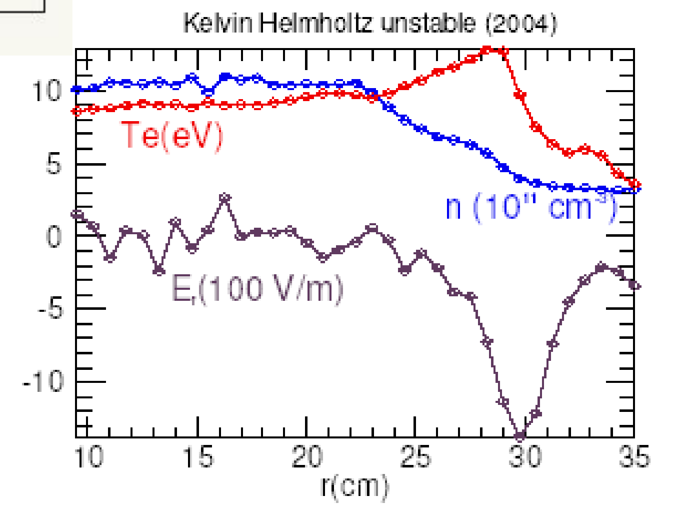
Interchange modes

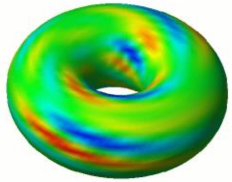


Vorticity Measurement on LAPD at UCLA



	KH	DWKH	DW
E_r	~ 1.5 kV/m	~ 0.4 kV/m	Weak
v_E	~ 10 km/s	~ 5 km/s	Slow
ρ_s	~ 10 mm	~ 5 mm	same
v_{de}	~ 0.6 km/s	~ 1.5 km/s	same





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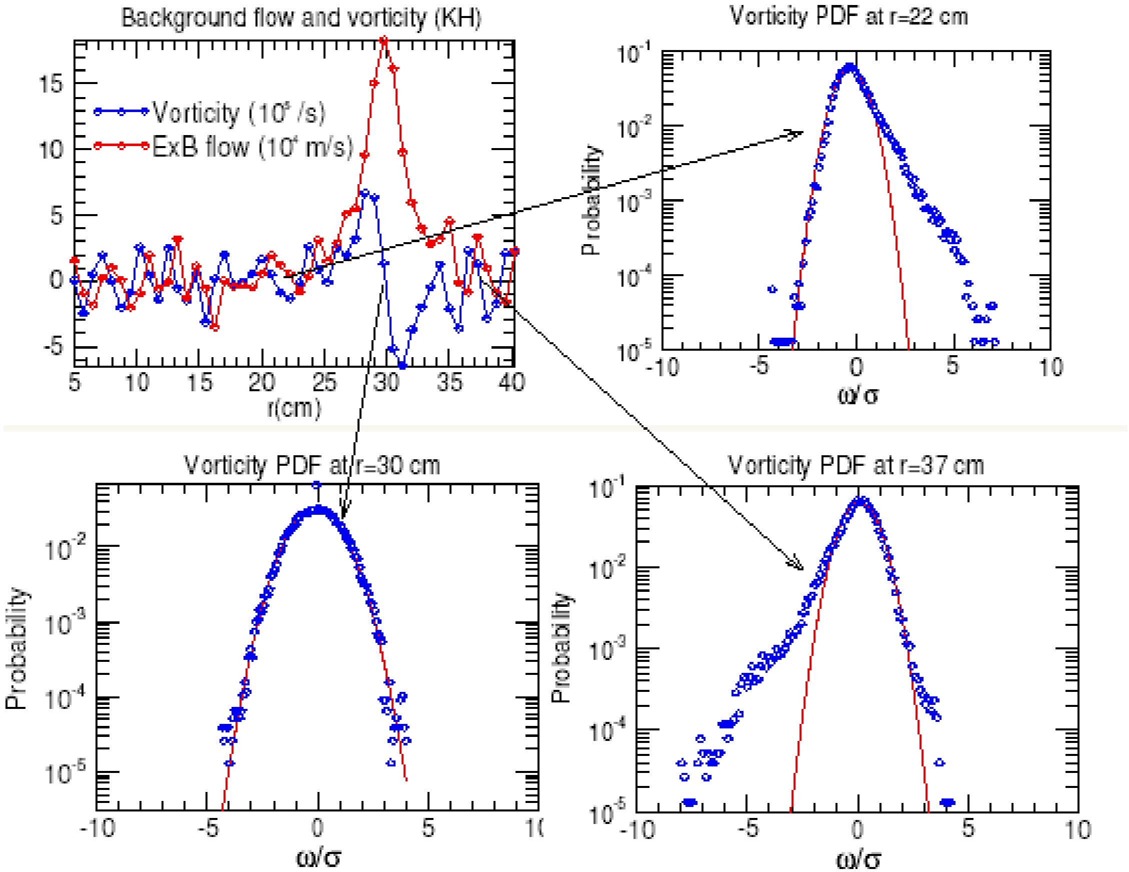
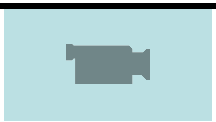
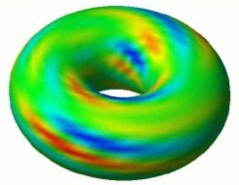


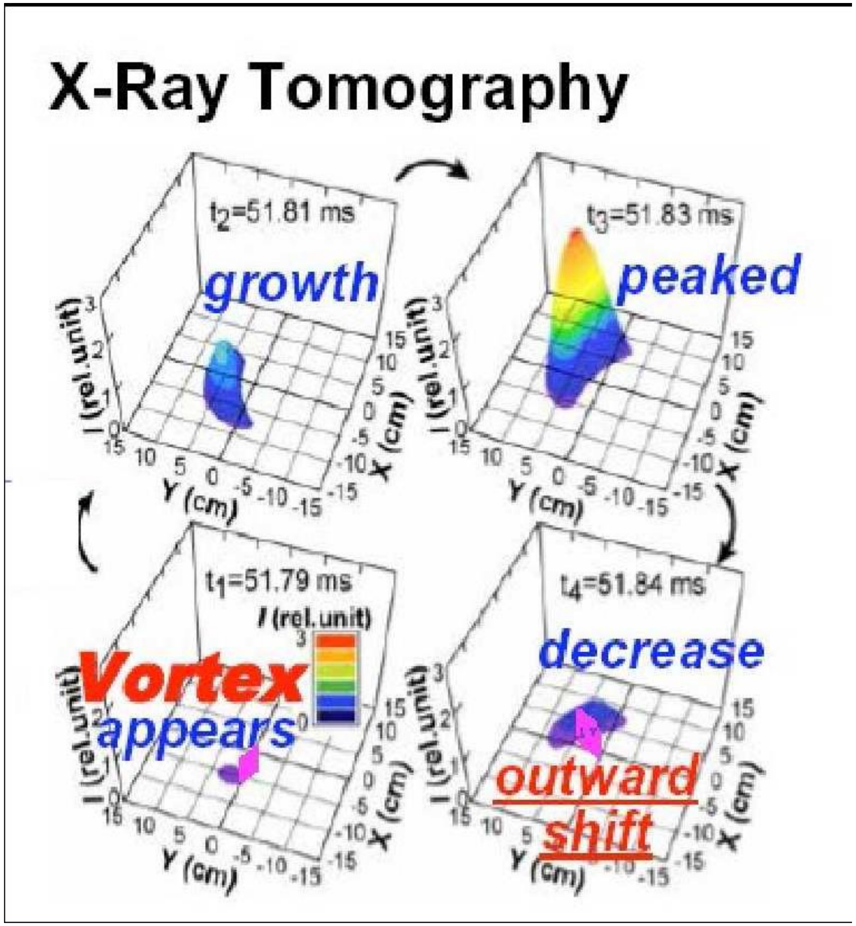
Electrostatic potential and vorticity in LAPD

Density and vorticity in LAPD or G10

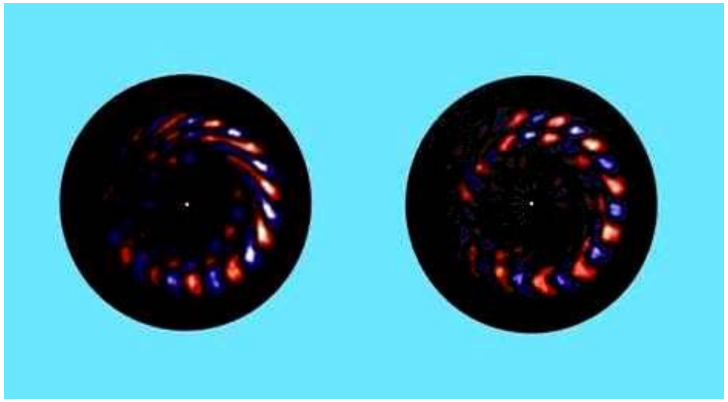




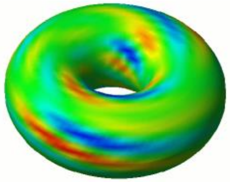
Soft X-Ray Tomography of Vortices in Gamma-10



- Vortices observed for the first time with fast time resolution
- E_r control of vortices in images and in simulations
- Kishimoto-Horton-Tajima PIC full torus/cylinder PoP 1996



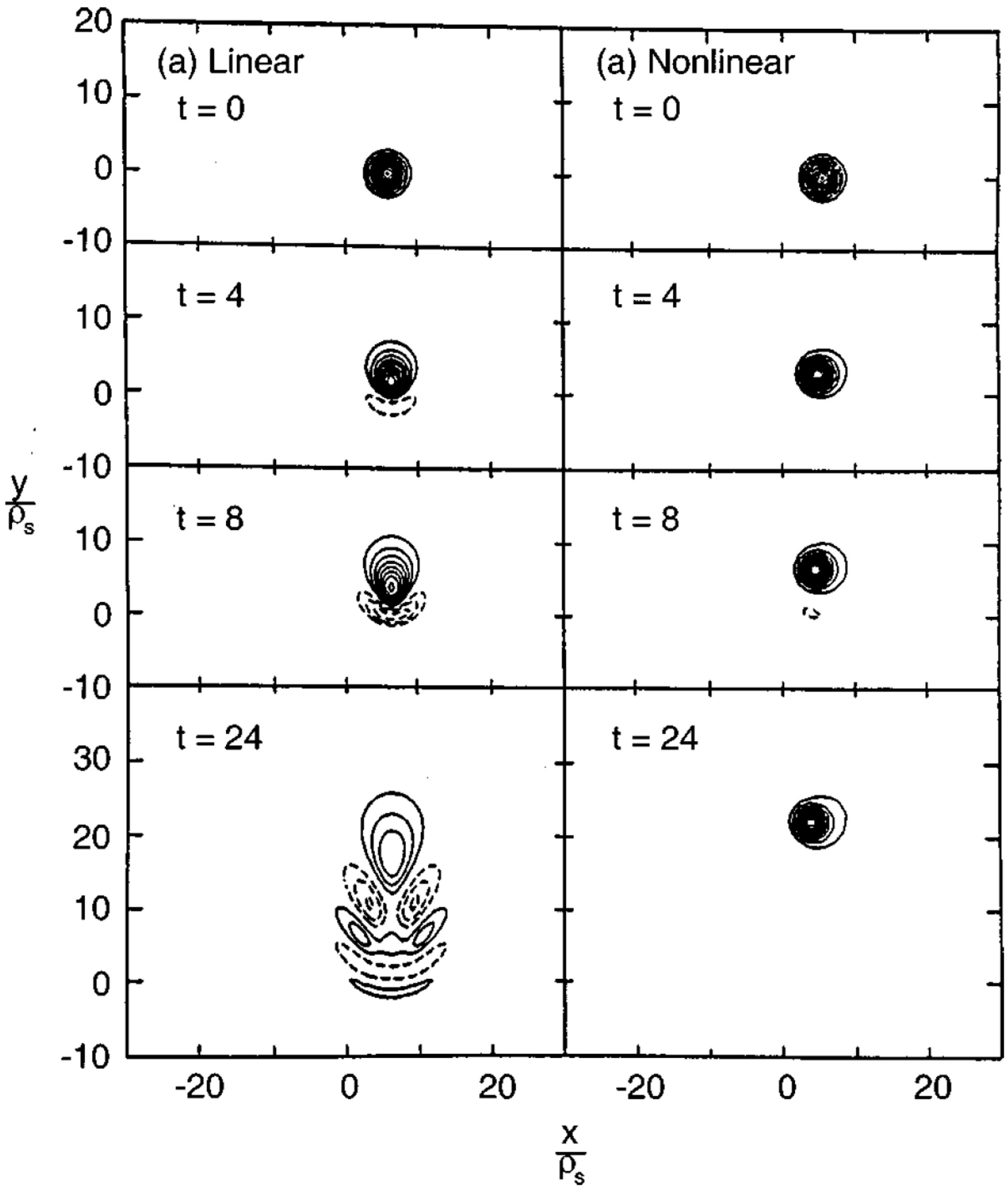
Two microchannel plates with CT reconstruction software. [T. Cho et al., Nucl Fusion (2005)]

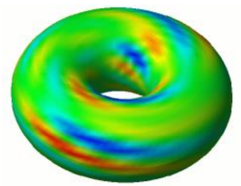


Nonlinear Self-focusing in EXB

- Left column shows the linear evolution of the disturbance with spreading from wave dispersion.

- Right column shows the self focused large amplitude vortex structure.





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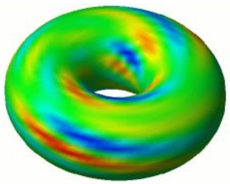
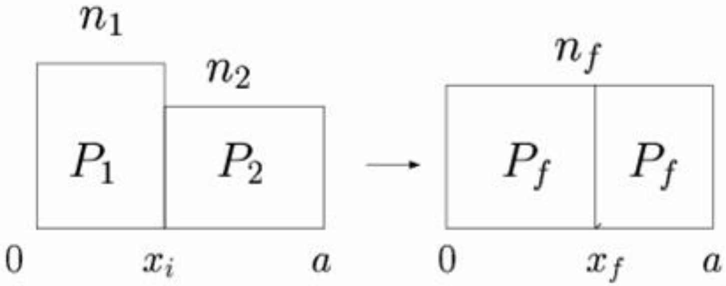
- Conditions for transport and propagation of disturbances
- Drift-wave as coherent structures including vortices and streamer



Energy Released by Drift Wave

T = const



$$W_{\rm turb} < W = \int_{x_i}^{x_f} (P_1 - P_2)\, dx A$$

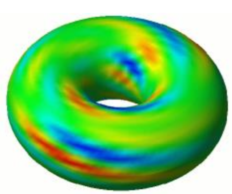
x_f is determined by

$$P_f = P_1 \left(\frac{x_i}{x_f}\right)^{\Gamma} = P_2 \left(\frac{a - x_i}{a - x_f}\right)^{\Gamma}$$

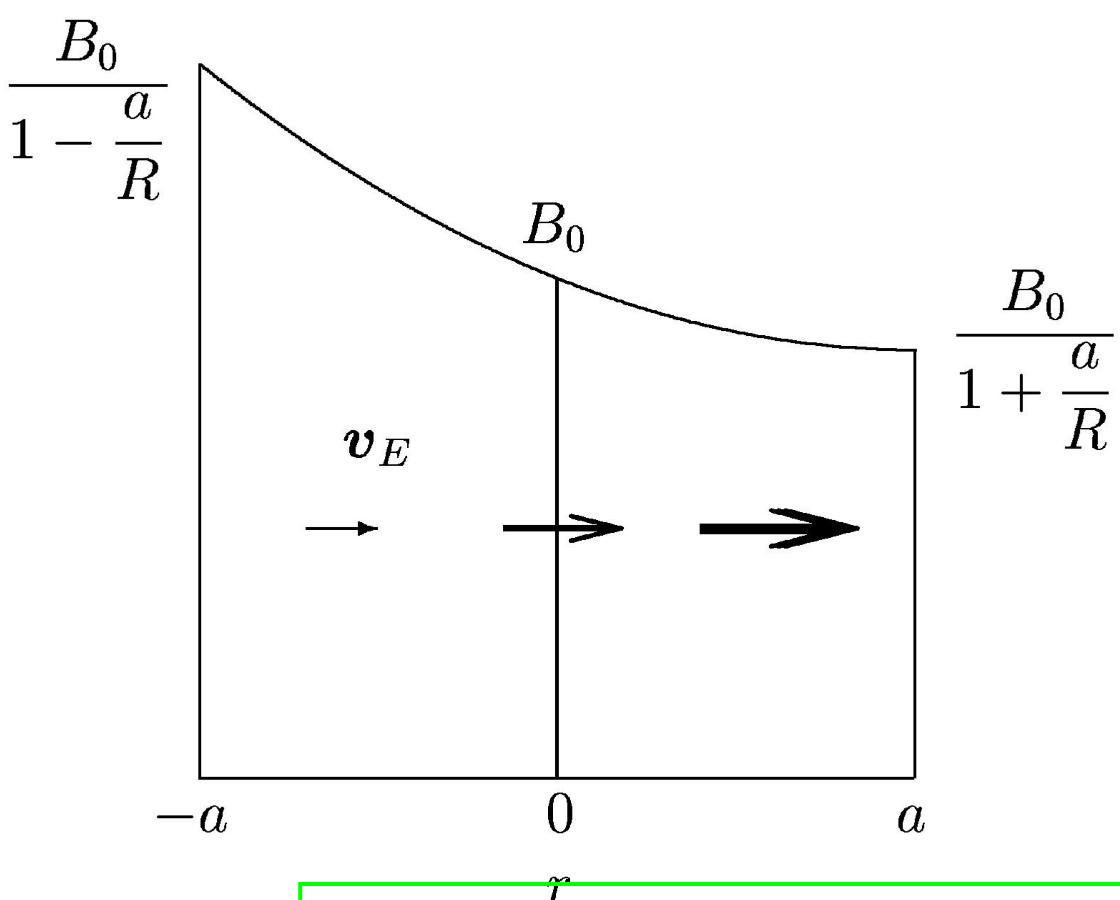
After some algebra,

$$W_{\rm turb} < W_{max} = \frac{(\Delta P)^2}{\Gamma P} x_i (a - x_i) \leq \frac{(\Delta P)^2}{4\Gamma P} a^2$$

$$W_{\rm turb} \;=\; \frac{1}{2} \min \langle v_E^2 \rangle < W_{max} \quad \to \quad \langle v_E^2 \rangle \leq \left(\frac{T_e}{eBL_n}\right)^2 = v_{de}^2$$



Compression/Refraction in Convection



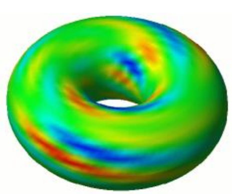
$$\nabla \cdot \boldsymbol{v}_E = \frac{2v_x \cos\theta}{R}$$

$$\frac{dp}{dt} = -\Gamma p \frac{2v_x \cos\theta}{R}$$

$$\epsilon_n = \frac{L_n}{R} \qquad \epsilon_p = \frac{L_p}{R}$$

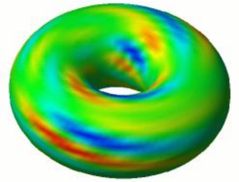
Toroidal ITG models e.g.~ Li & Kishimoto, PoP 02,04, Dong, et al. PoP 2003

Toroidal ETG models e.g. ~ Holland and Diamond, 2002, PoP 9, p. 3857. also, 2004 and 2005.



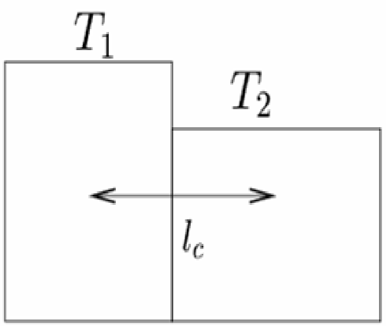
Anomalous Thermal Transport

- Electron transport is a phenomenon with its own characteristics
  - Kadomtsev – “Tokamak Plasma: A Complex Physical System” (1992) with large anomaly factor up to 100
- Ion thermal transport is typically anomalous with smaller anomaly over neoclassical transport across many confinement geometries
- Two different k,ω regimes of turbulence

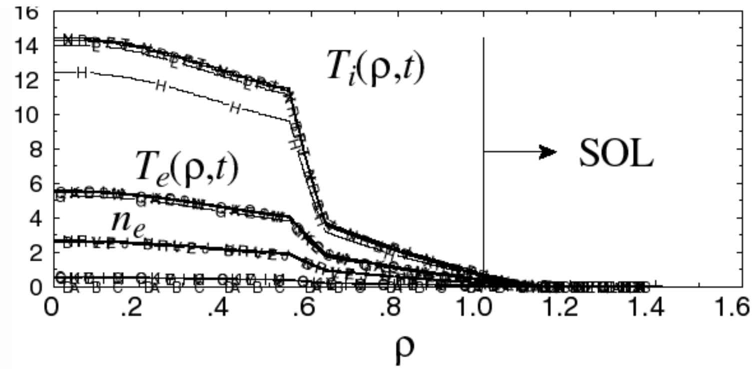


Carnot Cycle gives Energy Released from Temperature Gradients

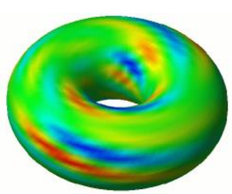
Carnot cycle gives maximum $W_{\rm turb}$ per cycle



$$
\begin{aligned}
W_{\rm turb} &= \Delta W_{\rm Carnot} = \Delta S \Delta T \\
\Delta S &= n\left(\frac{3}{2}\frac{\Delta T}{T} - \frac{\Delta n}{n}\right) \\
\Delta W_{\rm Carnot} &= \frac{3}{2} n T \frac{\Delta T}{T}\left(\frac{\Delta T}{T} - \frac{2}{3}\frac{\Delta n}{n}\right) \\
&= \frac{3}{2} n T l_c^2 (\partial_r \ln T)\left[\partial_r \ln T - \frac{2}{3}\partial_r \ln n\right]
\end{aligned}
$$



Critical gradient in fundamental limit without Landau damping η_{crit}=dln T/dln n = Γ-1 (=2/3 for Γ = 5/3)
Γ= adiabatic gas constant.



Ion Temperature Gradient Model

Hamaguchi-Horton Model

$$\frac{\partial \delta n_i}{\partial t} = -v_{de}\frac{\partial \delta\phi}{\partial y} - \nabla_{\|}\delta v_{\|} - \frac{c_s^2}{\Omega_i}\left[\delta\phi, \delta n_i\right]$$

$$\frac{\partial}{\partial t}\left(\delta n_i - \rho_s^2\nabla_\perp^2\delta\phi\right) = -v_{de}\frac{\partial}{\partial y}(1-\tau(1+\eta_i)\rho_s^2\nabla_\perp^2)\delta\phi - \nabla_{\|}\delta v_{\|} - \frac{c_s^2}{\Omega^i}\left[\delta\phi, (\delta n_i - \rho_s^2\nabla_\perp^2\delta\phi)\right]$$

$$\frac{\partial \delta v_{\|}}{\partial t} = -\frac{1}{m_i n_0}\nabla_{\|}\delta p_i - c_s^2\nabla_{\|}\delta\phi - \frac{c_s^2}{\Omega_i}\left[\delta\phi, \delta v_{\|}\right]$$

$$\frac{\partial \delta p_i}{\partial t} = -v_{de}\tau(1+\eta_i)\frac{\partial \delta\phi}{\partial y} - \gamma p_{i0}\nabla_{\|}\delta v_i - \frac{c_s^2}{\Omega_i}\left[\delta\phi, \delta p_i\right]$$

where $\tau = T_i/T_e$ and $v_{de} = (\rho_s/L_n)c_s$.

key driving term of dT_i/dx

Balescu, 2005, Aspects of Anomalous Transport in Plasmas, IoP pp.33-38.

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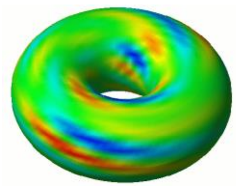
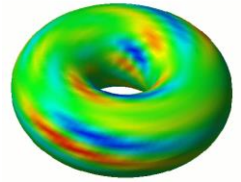


III. Drift-Wave Eigen-modes in Toroidal Geometry

- Ballooning modes from unfavorable magnetic curvature
- Trapped Particle Instability and the TEM mode
- Reversed magnetic shear and E_r shear modes



Magnetic Curvature Driven Interchange

Pressure Gradient Interchange

$p' \equiv -\frac{p}{L_p}$ and $\boldsymbol{E}_\perp = -\boldsymbol{v} \times \boldsymbol{B}$

$\gamma^2_{\mathrm{MHD}} = -\frac{p'}{\rho}\frac{2}{R}$

blobs accelerate out

$g_{\mathrm{eff}} = 2c_s^2/R$

$\sim 2(3 \times 10^5 \mathrm{m/s})^2/(3\mathrm{m})$

$\sim 6 \times 10^{10}\ \mathrm{m/s}^2$

over outside of torus

g_{eff} sets limits on three scales:

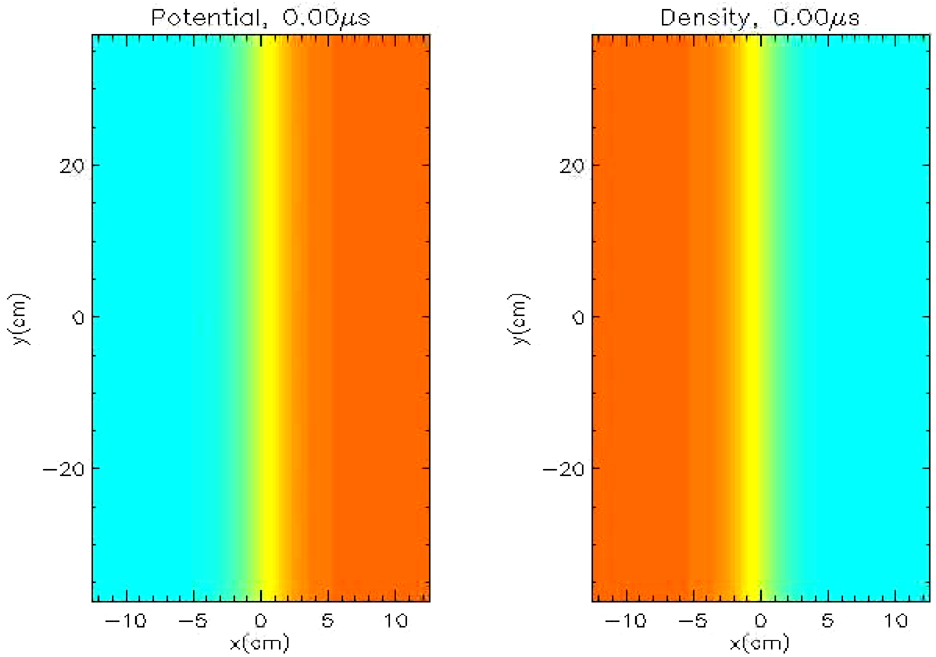
MHD p', ρ_{i}-ITG and ρ_{e}-ETG.

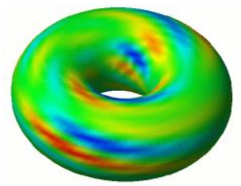
Rayleigh-Taylor

gravity g and

density gradient

$$\gamma^2_{\mathrm{RT}} = -\frac{g}{\rho}\frac{dp}{dr} = -\frac{\boldsymbol{g}\cdot\nabla\rho}{\rho}$$





$E_{\|} \neq 0$ Drift Waves Unstable from dn/dx

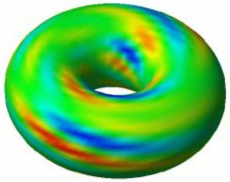
- Kadomtsev and Pogutse discvers Trapped Electron mode.
- Mirror trapped electrons in low-B region respond as though $k_{\|} = 0$ forming a shielded interchange mode on the outside of torus.

$$\frac{\tilde{n}_e}{n_e} = \left[1 - f_{\rm tr}\left\langle\frac{\omega - \omega_{*e}(1+\eta_e(\epsilon - 3/2))}{\omega - \omega_{De} + i\nu_{eff}/\epsilon^{3/2}}\right\rangle\right]\frac{e\phi}{T_e}$$

$$\frac{\tilde{n}_i}{n_i} = \left(\frac{\omega_{*e}}{\omega} - k_\perp^2\rho_s^2\right)\frac{e\phi}{T_e}$$

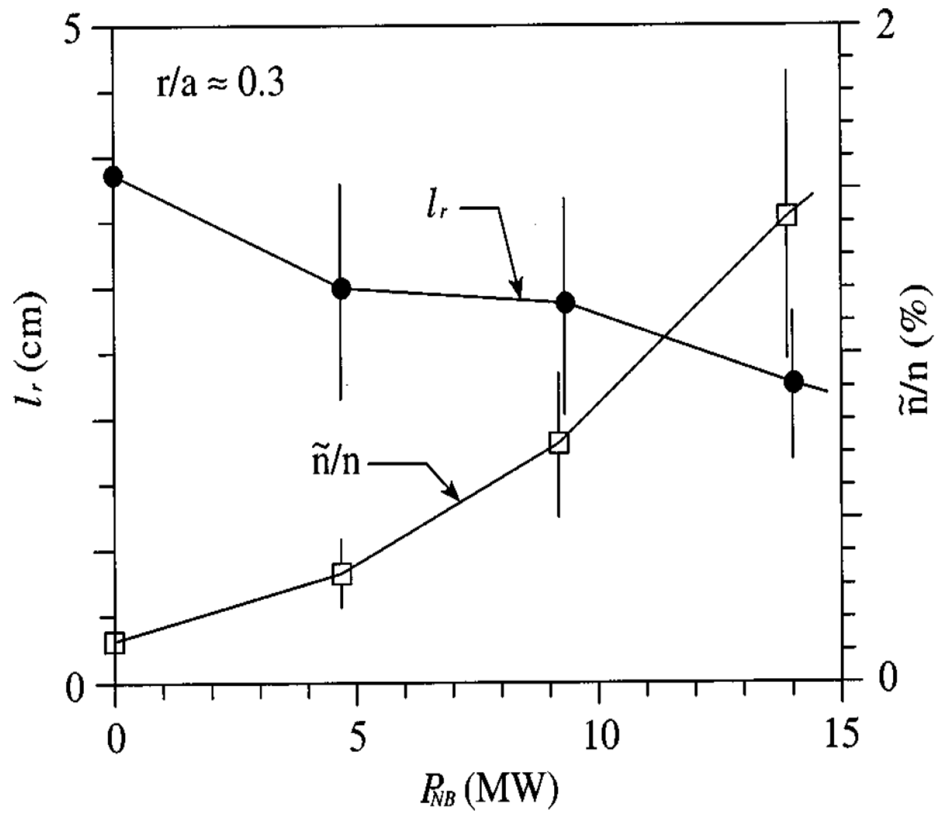
where trapped fraction of electron $f_{\rm tr}$, ion inertial scale length $\rho_s = (m_i T_e/eB)^{1/2} = (T_e/T_i)^{1/2}\rho_i[\sim \text{ cm}]$.

Source for TEM Refs: Wesson, Tokamaks 3, Oxford, p. 430 -441.

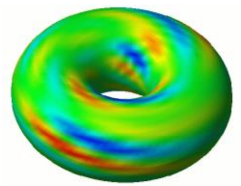


E&M (μ-wave) Scattering off $\delta n_e(r,t)$

- ✓ Mazzucato et al ATC and TFTR
- ✓ Brower et al TEXT and DIIID
- ✓ Zhu, Hoang et al Tore Supra
- ✓ Many E&M scattering experiments demonstrated the drift wave turbulent spectra typical spectral $k^{-3\sim4}$ and anisotropic in k_x-k_y plane



Mazzucato and Nazikian,1996

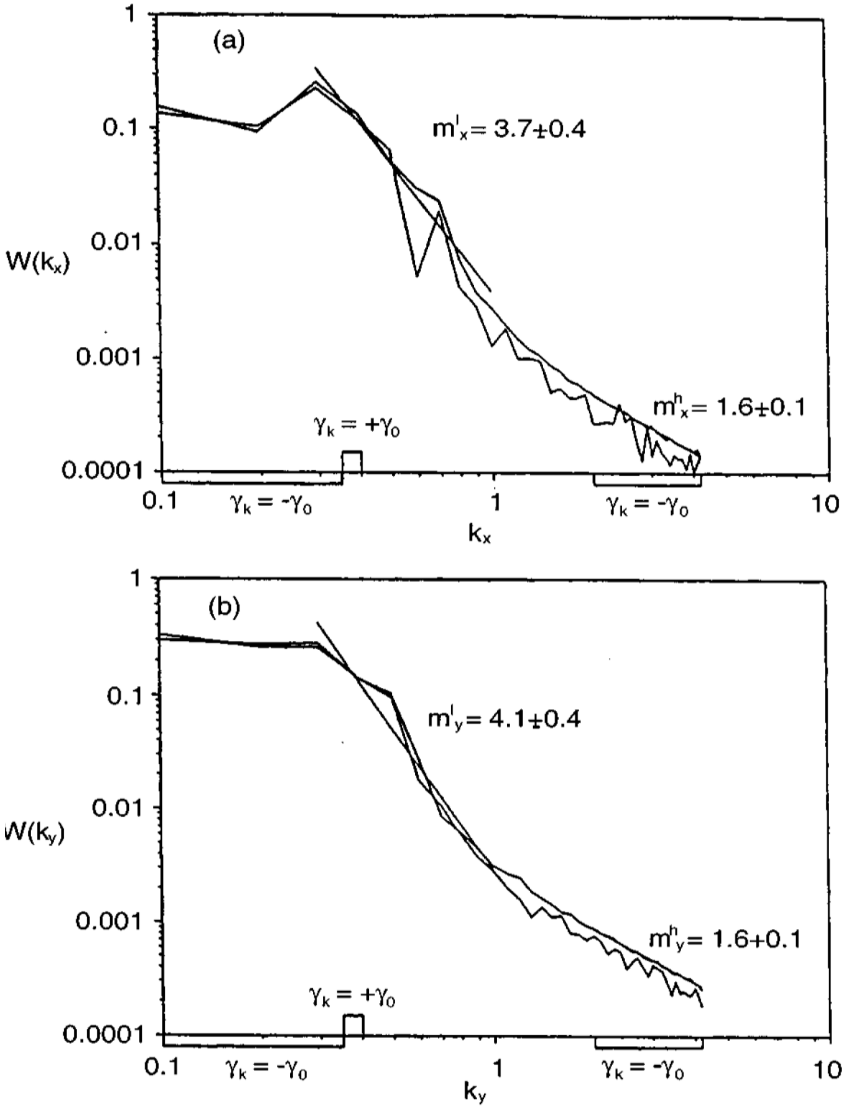


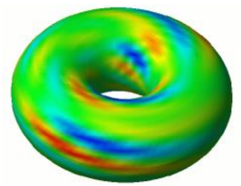
Spectral Exponents

- Weakly anisotropic and the spectral exponents for model problems with localized source γ_k of fluctuations

- At low k there is a transformation of fluctuations to zonal flows and streamers.

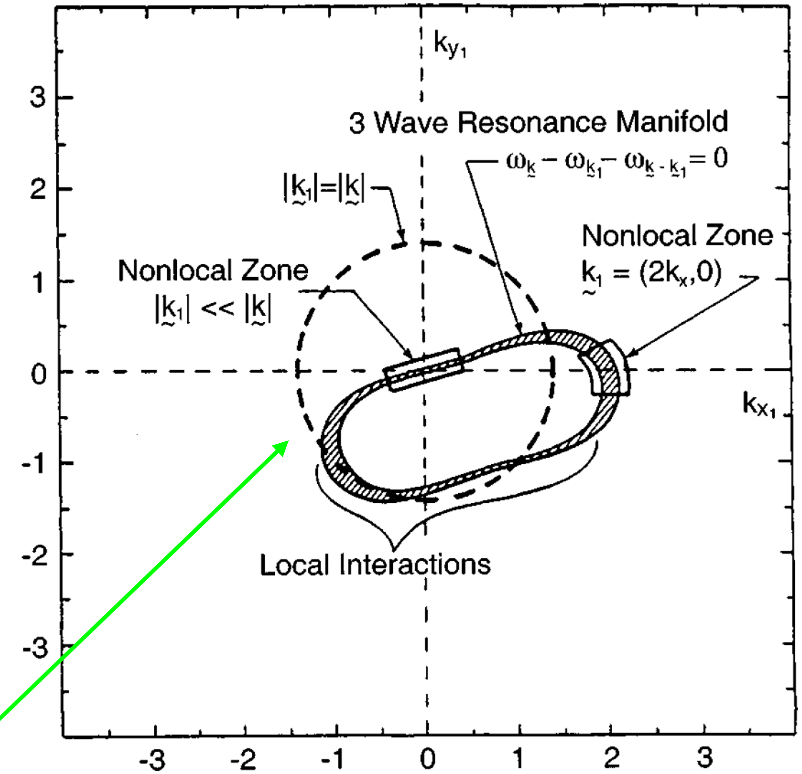
- Streamers have secondary instabilities.





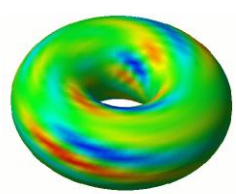
Three Wave Interactions in k-space

- Weak Turbulence Theory of Drift Waves
- Local and non-local interactions in k
- ➢ Lead to creation of streamers and zonal flows



Each (k_x, k_y) gives a resonant curve in $p=k_1$

$$\Delta\Omega_{\mathbf{k},p} = \frac{p_y v_d}{1+p_x^2+p_y^2} + \frac{(k_y - p_y)v_d}{1+(k_x-p_x)^2+(k_y-p_y)^2} - \frac{k_y v_d}{1+k_x^2+k_y^2}$$



Weak Turbulence Theory

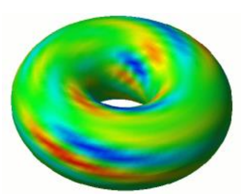
- Turbulence equations

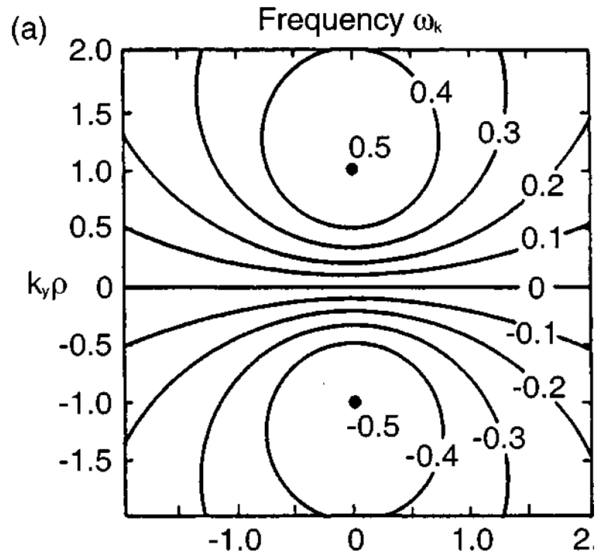
$$\frac{dW_k}{dt} = 2\gamma_k W_k - \sum_{k_1} (k \times k_1)^2 R_{k_1,k-k_1} \left[\left(k^2 - k_1^2\right) W_k W_{k_1} + \ldots \right]$$

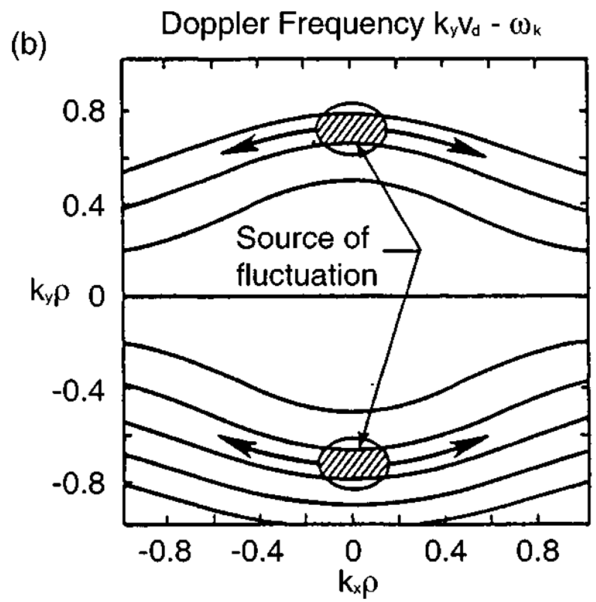
Benkadda, Doveil, Elskens, *Transport,Chaos & Plasmas*, 1996, ISBN 981-02-2696-9

✓Wave-Kinetic Equations conserve energy and momentum of wave quanta N_k

✓Action invariants and cascades.



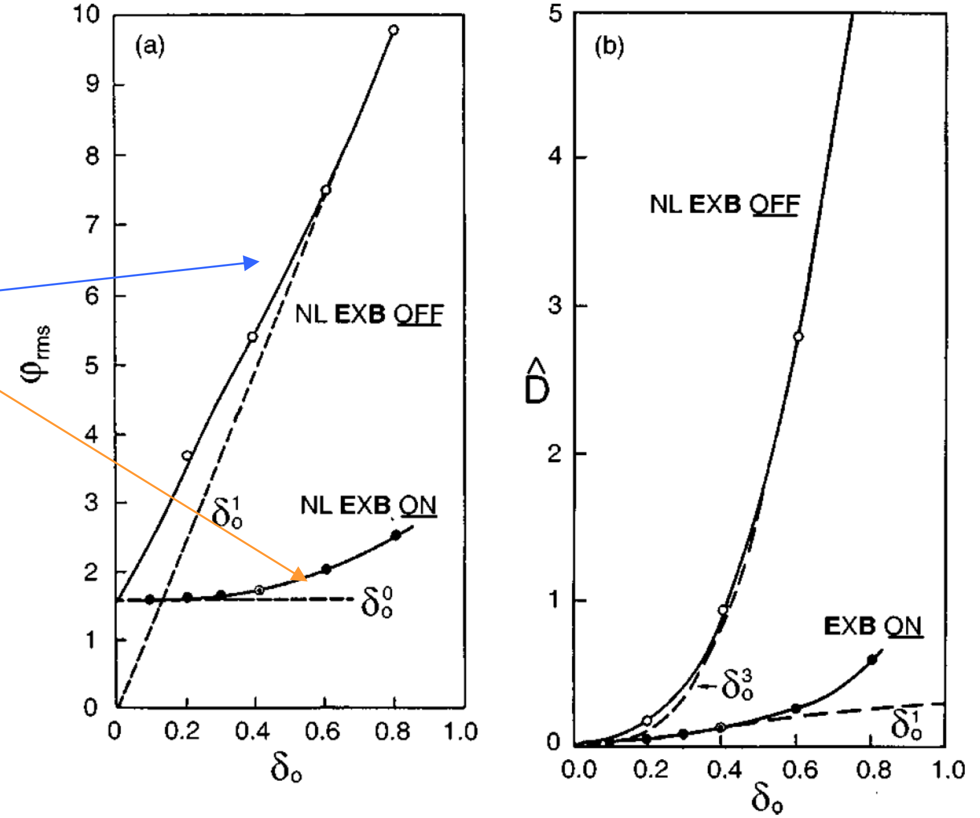


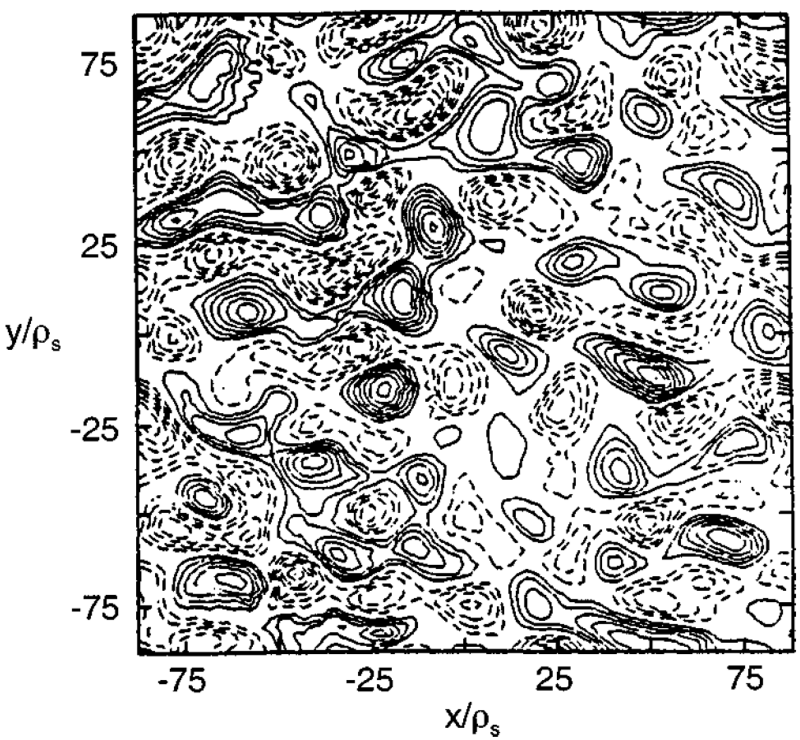


Saturation Levels in TEM modes

Terry-Horton Model with $i\delta_k$ non-adiabatic electron response 1983

Waltz simulation of TEM from GLF codes showing difference in H-M and Terry-Horton models



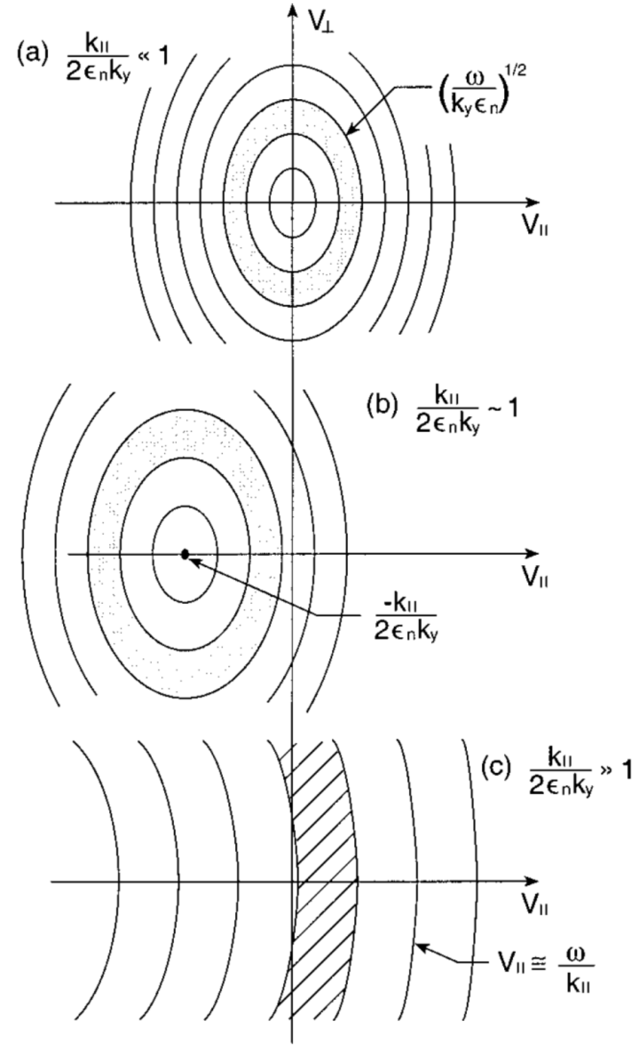


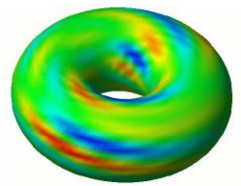
Typical turbulence φ-field

Kinetic Wave-Particle Resonance in Torus

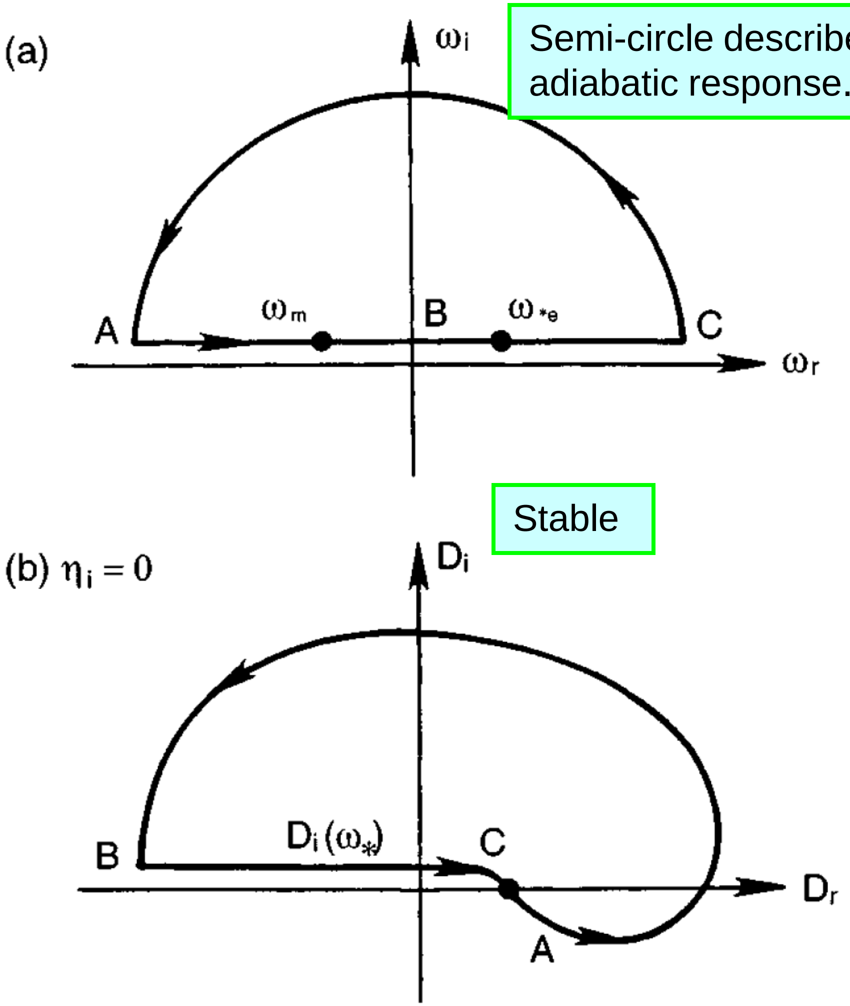
- The Landau resonance for drift waves in the torus is for particles $v_\perp$ and $v_\parallel$ such that

$$\omega = k_\parallel v_\parallel + v_D\left(\frac{1}{2}v_\perp^2 + v_\parallel^2\right)$$



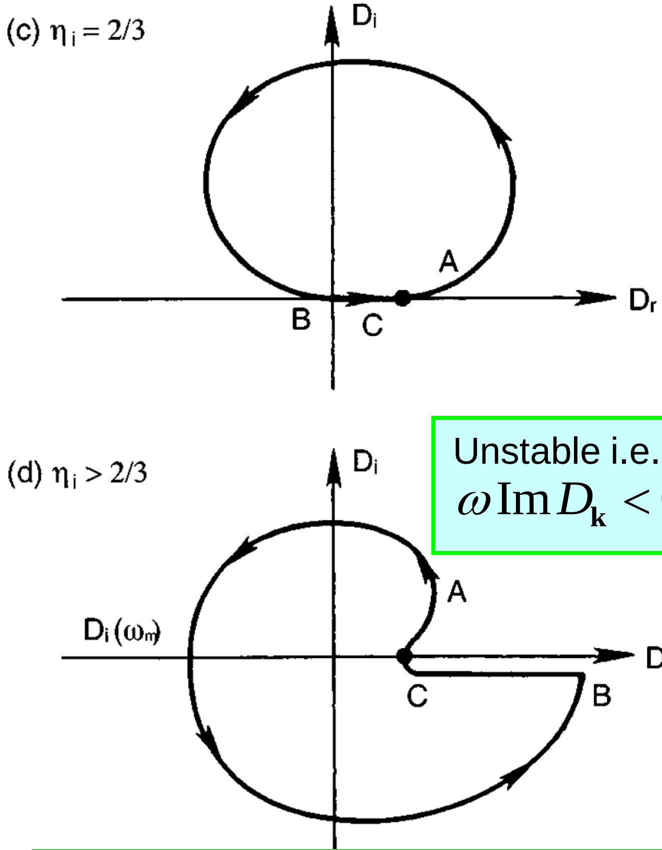


Nyquist Plot of D(k,ω) gives Critical ΔT



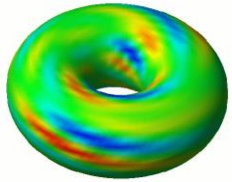
Semi-circle describes adiabatic response.

Stable

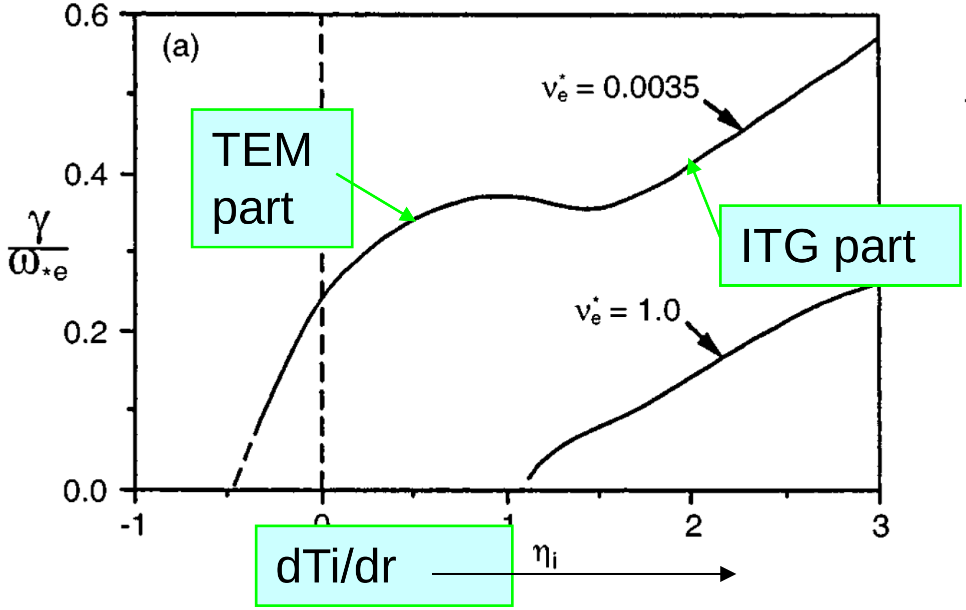


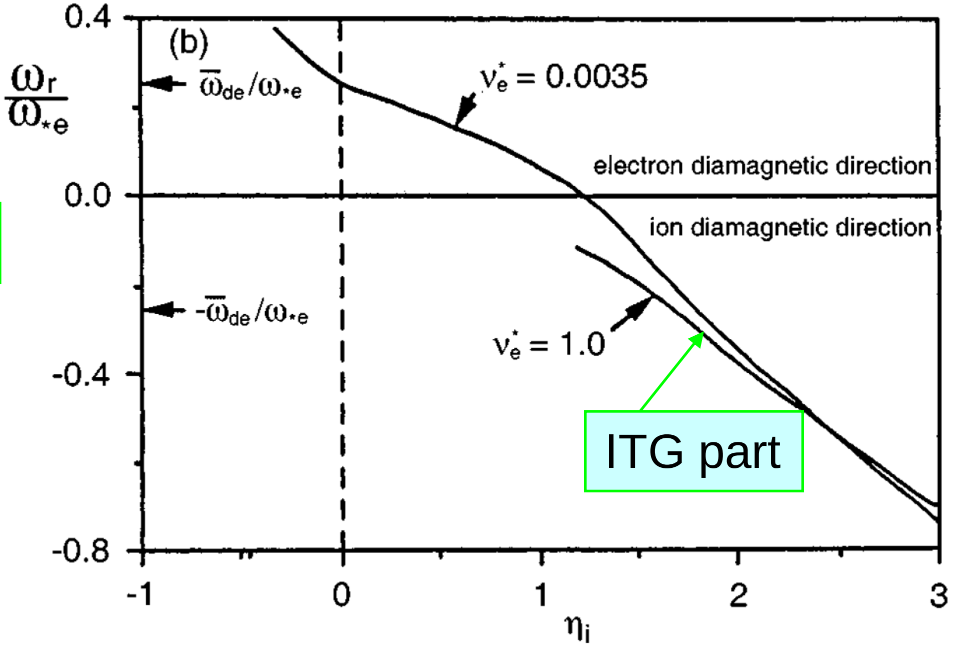
Unstable i.e. $\omega \, \mathrm{Im} D_{\mathbf{k}} < 0$

Details of stability analysis by Romanelli Phys. Fluids B (1989), Hong *et al* Phys. Fluids B (1989), Nordman & Weiland Nucl. Fusion (1989, 1990) and Kim *et al* Phys Plasmas (1994)

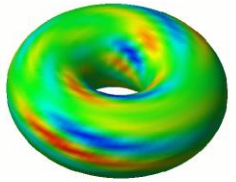


Typical Growth rates in TEM-ITG Modes



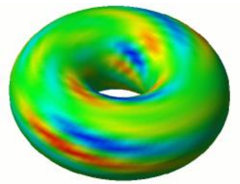


- Rewoldt-Tang (1990 Phys Fluids B) general toroidal eigenmode code with basis functions from sum of local sheared eigenmodes
  - complex harmonic oscillators on rational surfaces .

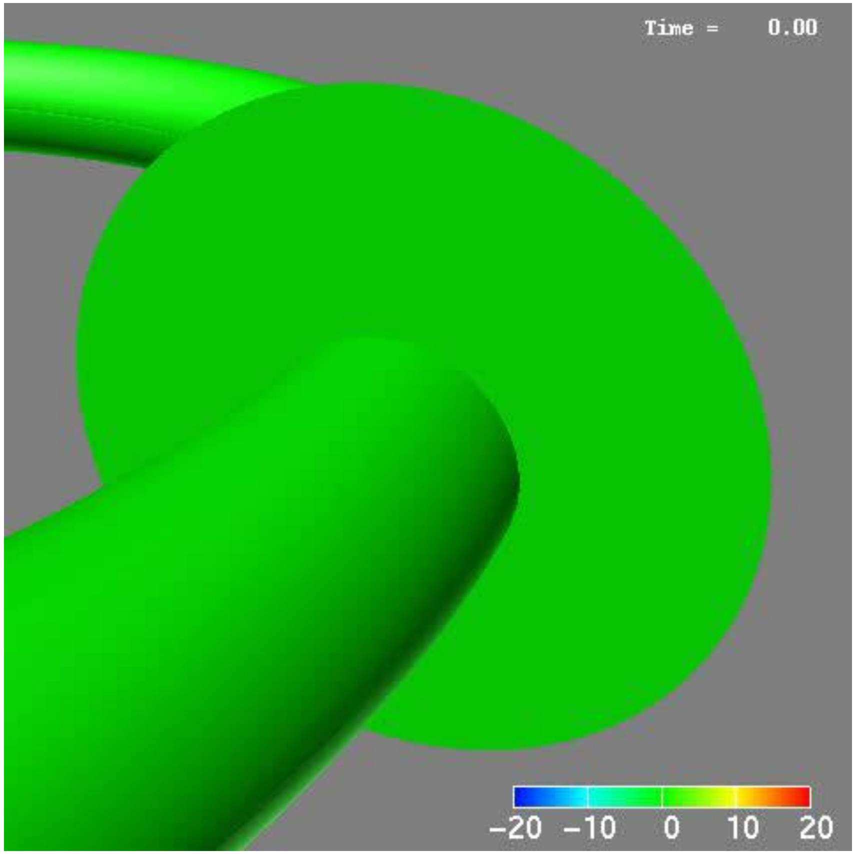


IV.Turbulence Simulations

- Inverse cascade and generation of sheared flows
- Ion acoustic wave coupling and dispersion from the polarization current
- Ballooning modes and turbulence simulations



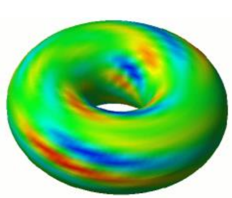
Ion Temperature Gradient in Tokamak



ITG turbulence electrostatic potential perturbations (blue > 0, red < 0)

- ✓ Radially elongated structures are quickly destroyed by zonal flows
- ✓ In the statistical steady-state turbulence and zonal flows co-exist
- ✓ High resolution in wave-number space due to a large number of toroidal modes (n=1 to 84)

T.-H. Watanabe and H. Sugama
AIP Proceedings & NF 2006 p.24



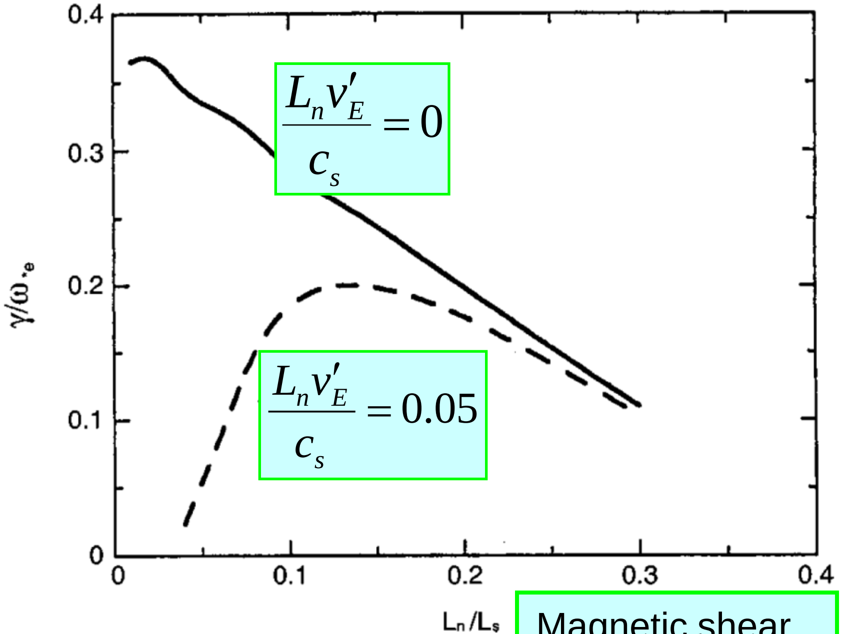
Cyclone DIII-D Parameters:

η_i= 3.1 r_0/R_0 = 0.18,
q_0 = 1.4, s = 0.8,
R_0/L_T = 6.92,$v_i L_n/v_t$=0.001

Parker, Lee 1993, Waltz-Krebel 1994, LeBrun et al 1993

Examples of Sheared Flow Transport Suppression

Gyro-kinetic Growth rate



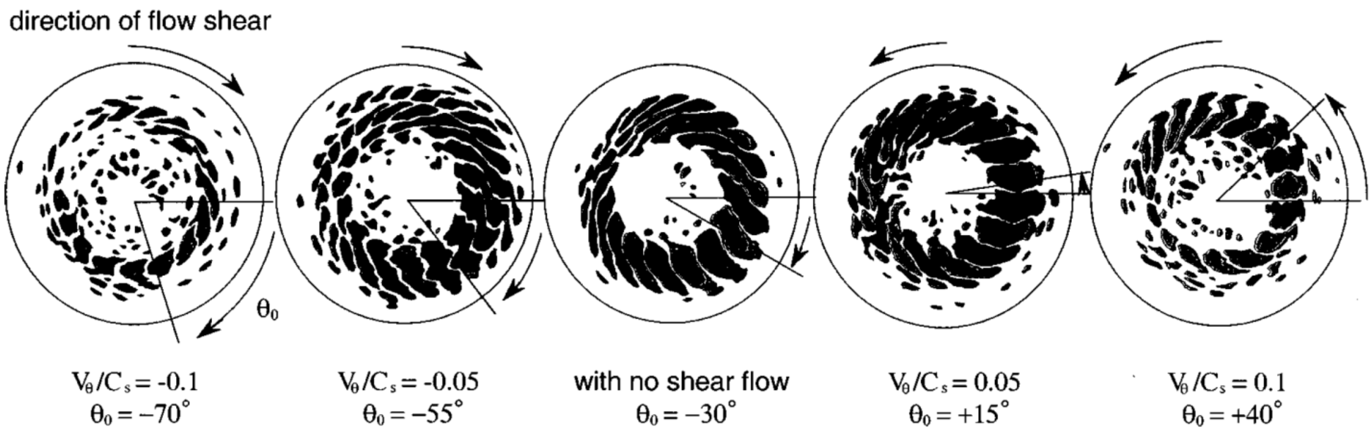
Magnetic shear

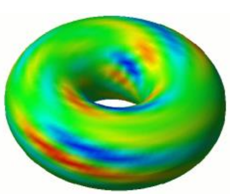
Kishimoto et al, NF 2000, 2005 PRL 2002
Dong et al. 2002,2004
P. H. Diamond Malkov, 2001
Diamond, Champeaux, Malkov NF 2001
Diamond and Smolyakov, PoP 2001,1999
E. Kim and P. H. Diamond, PoP 2002 and PRL2003
Newman, Carreras, Diamond, Hahm PoP 1996

Electron drift clockwise

Change of global drift-wave functions w.r.t $E_r(r)$

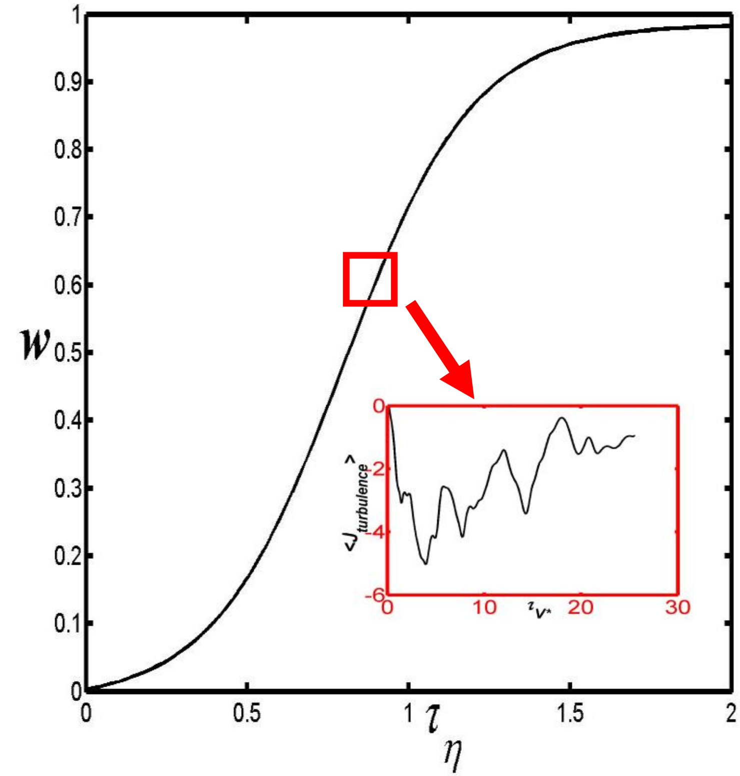
Kim *et al* Phys Plasmas (1996





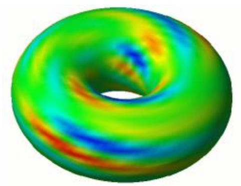
Interaction Between Magnetic Islands and Electrostatic Turbulence – New Problem & Issue

Science Question: How does the turbulent transport proceed in high beta advanced tokamaks where there are significant magnetic islands on t



1) Physical model: generalize the Hasegawa-Wakatani drift wave model for plasma with rotating islands.
2) What happens to the turbulence in the island and the transport across the island?
3) Separation of the time scales allows an electrostatic approach with fixed magnetic islands.

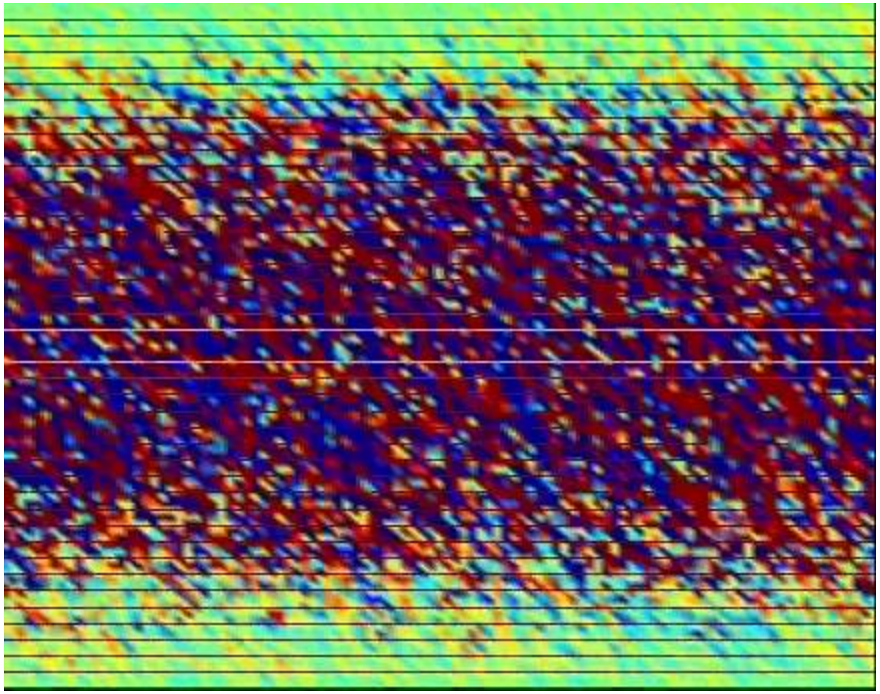
Militello and Waelbroeck

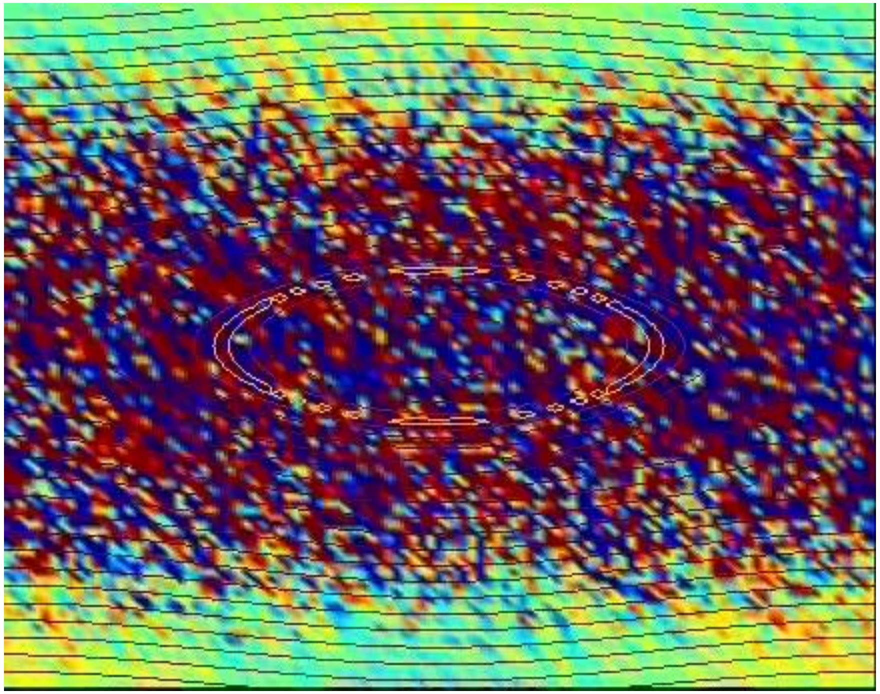


Comparison of Drift Turbulence with and without a Magnetic Island

Militello and Waelbroeck

Sheared Magnetic Field **Magnetic Island**

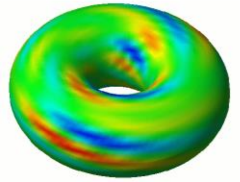




V. Transport Simulations for large Tokamaks with Turbulence

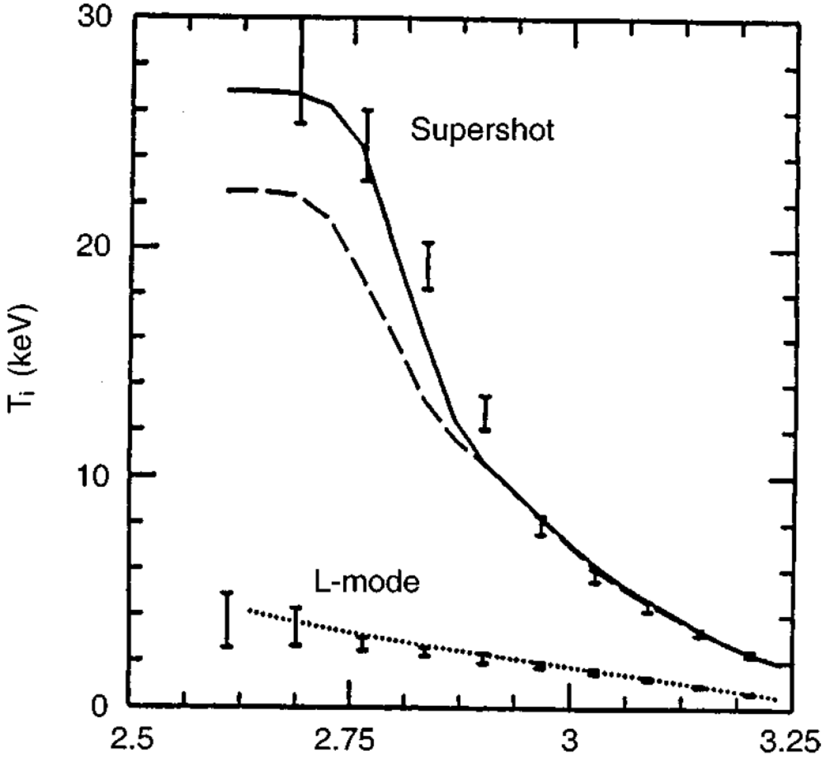
Simulation examples for TFTR

JT 60U and Tore Supra



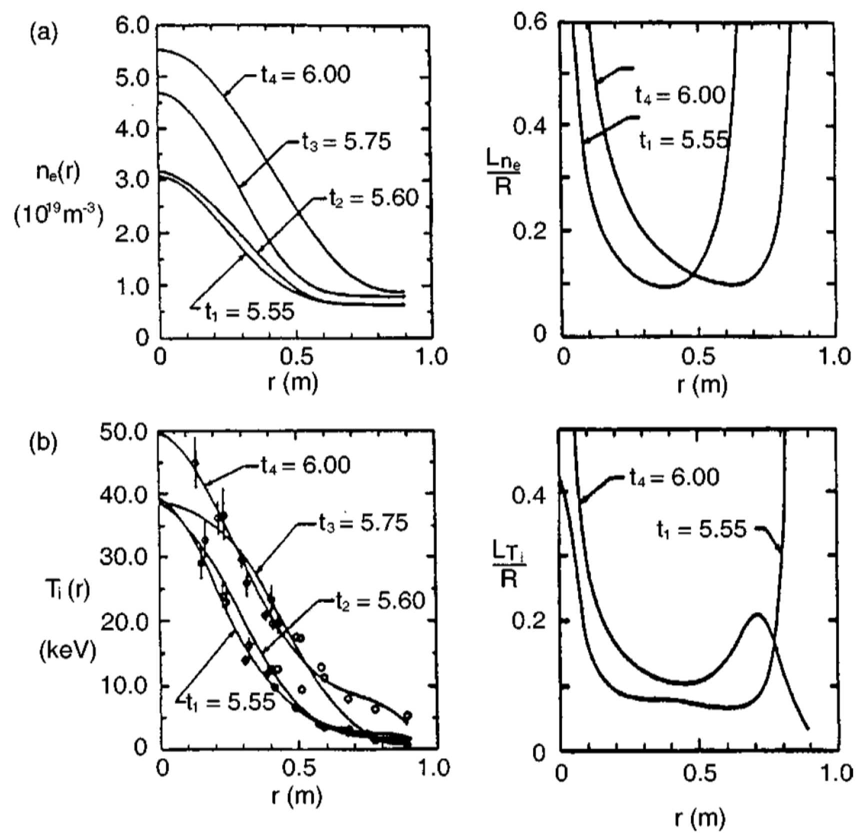
TFTR Super-shot and Koide et al JT60-U

TFTR L-mode and Supershot

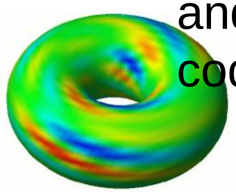


Transport Barrier in T_i with modeling by Kotschenreuther and Dorland et al GS & GS2 codes

JT 60U Internal Transport Barrier in ITG simulation



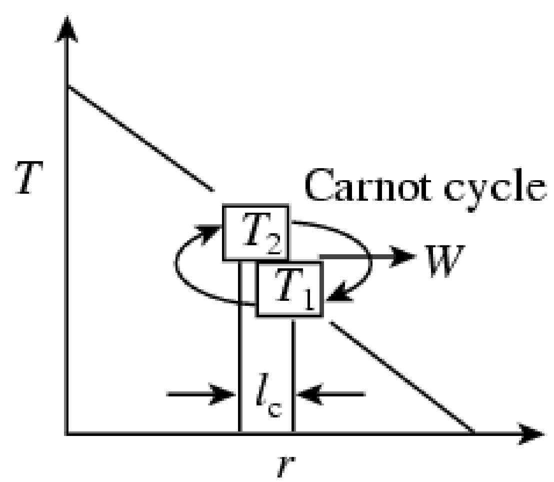
Transport Barrier in ITG from and sheared flows



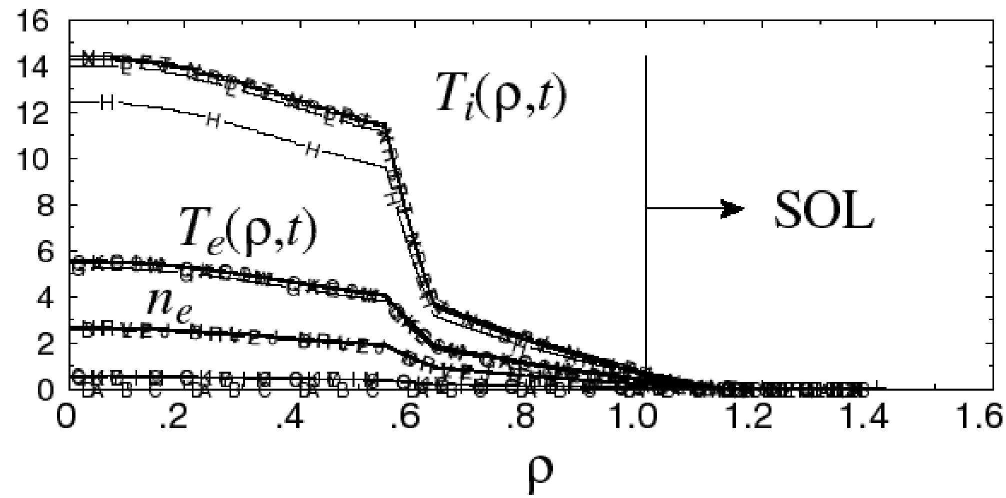
Energy Release from Carnot Cycles

- Temperature Gradient driven turbulence

Threshold ∇T for Convection

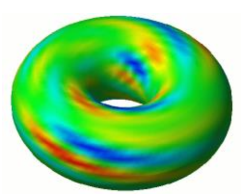


Model of Internal Transport Barrier for JT-60U Shot E27969



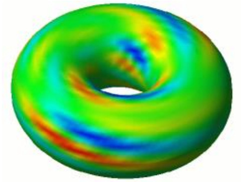
Record shot with D-T equivalent Q in D-D discharge of Q_{fusion} ~1

Reversed Magnetic Shear, Ishida et al. PRL, 1997.



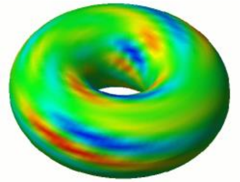
Conclusions, Challenges and Opportunities

- Electron and Ion Turbulent Transport
  - Physics of thresholds for Turbulence- still nonlinear issues
  - Carnot cycle for Critical Temperature Gradient - realtime tool?
- Nonlinear states with coherent structures [CS]: vortices, streamers and zonal flows in the sea of turbulence. -Challenge for future.
- Classical Models for density gradient driven drift waves.
- Classical Models for Ion Temperature Gradient (ITG) driven drift waves. The Standard Model of ITG-TEM transport. -Integration?
- Experimental observations of drift waves. High and low k issues.
- Gyrofluid simulations show direct and inverse cascades and CS.
- Ion scale turbulence explains Internal Transport Barriers and the empirical confinement scaling laws for hot ion plasmas.
- Many challenges and opportunities for new researchers!



The End
La Fin

ETG turbulence on Friday Morning



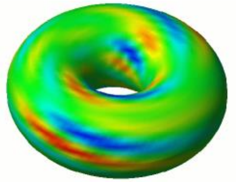
Interchange Modes and Shear Alfven Waves

- MHD interchange mode

$$\omega(\omega - \omega_{*i}) + \frac{2k_y^2}{k_\perp^2}\frac{p}{\rho R L_p} = 0 \quad \rightarrow \quad \gamma^2_{\rm MHD} = \frac{2p}{\rho R L_p}\,.$$

- Alfven Wave, with $k_\perp^2 \rho_s^2, k_\perp^2 \delta^2 \ll 1$, then, $A_\| = k_\| \phi/\omega \rightarrow E_\| = 0$,

$$\omega(\omega - \omega_{*i}) - \frac{k_z^2 B^2}{\rho \mu_0} = 0\,.$$

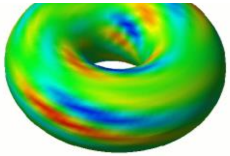


Pressure Limits for Tokamaks

β Limit of Tokamaks[a]

- β Limit in first stability region of ballooning mode
  - Large aspect-ratio approximation':
    $\beta_t \leq 0.3\frac{\epsilon}{q_0 q_a}\left(1 - \frac{q_0}{q_a}\right)$ [Freidberg 1987]
  - Sykes limit:
    $\beta_t \leq 0.044\left(\frac{I_0}{aB_0}\right)$ or $\beta_t \leq 0.22\left(\frac{\epsilon\kappa}{q_*}\right)$ [Sykes *et al.*, 1983]
    where units are $I_0(MA)$, $a(m)$, $B_0(T)$, and $\epsilon = a/R_0$, $q_* = 2B_0A/\mu_0R_0I_0$, A is the cross-sectional area, and $\kappa = A/\pi a^2$ is the elongation.
- β limit due to low-n external ballooning-kink mode
  - Troyon limit:
    $\beta_t \leq 0.028\left(\frac{I_0}{aB_0}\right)$ or $\beta_t \leq 0.14\left(\frac{\epsilon\kappa}{q_*}\right)$ [Troyon *et al.*, 1984]
- No β limit in second stability region of ballooning mode.

[a] Freidberg, *Ideal Magnetohydrodynamics*, 1987



V. Temperature Gradient Driven Drift Waves

- Properties of the ITG drift-wave instability: Comments on ETG
- Wave-particle power transfer and the Nyquist diagram
- Scaling laws of the ion temperature gradient turbulent transport